Earth Science Data & Formula Sheet

Constants and Earth Data

Gravitational constant	$G = 6.674 \times 10^{-11}$ N·m^2/kg^2	Standard gravity	$g = 9.81$ m/s^2
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}$ W/(m^{2}K^4)	Wien constant	$b = 2.898 \times 10^{-3}$ m·K
Universal gas constant	$R = 8.314$ J/(mol·K)	Avogadro number	$N_A = 6.022 \times 10^{23}$ mol^{-1}
Gas constant, dry air	$R_d = 287$ J/(kg·K)	Specific heat, dry air	$c_p = 1004$ J/(kg·K)
Molar mass, dry air	$M = 0.02897$ kg/mol	Mean sea-level pressure	$p_0 = 1013.25$ hPa
Latent heat, vaporisation	$L_v = 2.50 \times 10^6$ J/kg	Latent heat, melting	$L_f = 3.34 \times 10^5$ J/kg
Specific heat, water	$c_w = 4186$ J/(kg·K)	Solar constant	$S_0 = 1361$ W/m^2
Earth radius (mean)	$R_E = 6371$ km	Earth mass	$M_E = 5.972 \times 10^{24}$ kg
Earth's rotation rate	$\Omega = 7.292 \times 10^{-5}$ rad/s	Mean density of Earth	$\bar{\rho}_E = 5514$ kg/m^3
Ocean area	$A_O = 3.61 \times 10^{14}$ m^2	Planetary albedo	$\alpha \approx 0.30$
Density: fresh water / sea water	1000 / 1025 kg/m^3	Density: ice	917 kg/m^3
Density: continental / oceanic crust	≈ 2700 / 2900 kg/m^3	Density: upper mantle	≈ 3300 kg/m^3

1 Atmosphere and Weather

1.1 Ideal Gas and Hydrostatic Balance

$$p = \rho R_d T, \qquad \frac{dp}{dz} = -\rho g$$

$$p(z) \approx p_0\, e^{-z/H}, \qquad H = \frac{R_d T}{g} \approx 8 \text{ km}$$

p pressure [Pa], ρ air density [kg/m^3], T temperature [K], z height [m], H scale height

1.2 Lapse Rates

$$\Gamma_d = \frac{g}{c_p} \approx 9.8 \text{ K/km} \quad \text{(dry adiabatic)}$$

$$\Gamma_m \approx 4 \text{ to } 7 \text{ K/km} \quad \text{(moist adiabatic)}$$

$$\Gamma_{\text{env}} \approx 6.5 \text{ K/km} \quad \text{(standard atmosphere)}$$

1.3 Humidity

$$e_s(T) \approx 6.112 \text{ hPa} \cdot \exp\left(\frac{17.67\,T}{T + 243.5}\right)$$

$$\text{RH} = \frac{e}{e_s(T)} \times 100\%$$

e vapour pressure, e_s saturation vapour pressure, T in °C (Magnus formula); the dew point is the T at which $e_s(T) = e$

1.4 Rotation and Wind

$$f = 2\Omega \sin\varphi \quad \text{(Coriolis parameter)}$$

$$V_g = \frac{1}{\rho f}\frac{\Delta p}{\Delta n} \quad \text{(geostrophic wind)}$$

φ latitude, $\Delta p/\Delta n$ horizontal pressure gradient; in the Northern Hemisphere the wind blows with low pressure on its left

2 Radiation and Climate

2.1 Black-Body Radiation

$$F = \sigma T^4, \qquad \lambda_{\text{max}} = \frac{b}{T}$$

2.2 Planetary Energy Balance

$$\frac{S_0(1-\alpha)}{4} = \sigma T_e^4 \quad \Rightarrow \quad T_e \approx 255 \text{ K}$$

$$I = S_0 \cos\theta_z \quad \text{(top of atmosphere)}$$

T_e effective radiating temperature (observed mean surface temperature $\approx$ 288 K; the difference is the greenhouse effect), θ_z solar zenith angle

2.3 Radiative Forcing and Response

$$\Delta F_{\mathrm{CO_2}} \approx 5.35\ \mathrm{W/m^2} \cdot \ln \frac{C}{C_0}$$
$$\Delta T \approx \lambda\, \Delta F, \qquad \lambda \approx 0.8\ \mathrm{K/(W/m^2)}$$

C, C_0 $\mathrm{CO_2}$ concentrations (pre-industrial $C_0 \approx 280$ ppm), λ climate sensitivity parameter (equilibrium)

2.4 Climate Archives

$$\delta^{18}\mathrm{O} = \left(\frac{R_{\mathrm{sample}}}{R_{\mathrm{standard}}} - 1 \right) \times 1000\ \text{per mil}, \qquad R = \frac{^{18}\mathrm{O}}{^{16}\mathrm{O}}$$

Orbital (Milankovitch) cycles: eccentricity ≈ 100 kyr, obliquity ≈ 41 kyr, precession ≈ 19 to 23 kyr

3 Water, Oceans and Ice

3.1 Water Balance and Flow

$$P = ET + Q + \Delta S$$
$$Q = v\, A \quad \text{(discharge)}$$

P precipitation, ET evapotranspiration, Q runoff, ΔS change in storage, v mean flow speed, A cross-section

3.2 Groundwater

$$Q = -K A\, \frac{\Delta h}{\Delta L} \quad \text{(Darcy's law)}$$
$$v_{\mathrm{pore}} = \frac{Q}{A\, n_e}, \qquad n = \frac{V_{\mathrm{voids}}}{V_{\mathrm{total}}}$$

K hydraulic conductivity [m/s], $\Delta h / \Delta L$ hydraulic gradient, n porosity, n_e effective porosity

3.3 Ocean Waves

$$c = \sqrt{g\, h} \quad \text{(shallow water, also tsunamis)}$$
$$c = \frac{g\, T}{2\pi} \quad \text{(deep water)}$$

h water depth, T wave period

3.4 Sea Level from Melting Land Ice

$$\Delta h \approx \frac{\rho_{\mathrm{ice}}}{\rho_{\mathrm{water}}} \cdot \frac{V_{\mathrm{ice}}}{A_O}$$

Floating sea ice hardly changes sea level when it melts. Mean ocean salinity ≈ 35 g/kg

4 Biosphere and Soils

4.1 Photosynthesis and Respiration

$$6\,\mathrm{CO_2} + 6\,\mathrm{H_2O} \xrightarrow{\text{light}} \mathrm{C_6H_{12}O_6} + 6\,\mathrm{O_2}$$
$$\mathrm{NPP} = \mathrm{GPP} - R_a$$

GPP gross and NPP net primary production, R_a plant respiration; roughly 10% of the energy passes from one trophic level to the next

4.2 Grain Sizes (ISO 14688)

$$\text{clay} < 0.002\ \text{mm} < \text{silt} < 0.063\ \text{mm} < \text{sand} < 2\ \text{mm} < \text{gravel} < 63\ \text{mm}$$

4.3 Settling of Particles

$$v_s = \frac{2}{9}\, \frac{(\rho_p - \rho_f)\, g\, r^2}{\mu} \quad \text{(Stokes' law)}$$

r particle radius, ρ_p, ρ_f particle and fluid density, μ dynamic viscosity (water $\approx 1.0 \times 10^{-3}$ Pa·s)

4.4 Soil Horizons

O organic litter, A topsoil with humus, E eluviated, B subsoil (accumulation), C weathered parent material, R bedrock

5 Rocks, Minerals and Earth History

5.1 Radiometric Dating

$$N = N_0\, e^{-\lambda t}, \qquad \lambda = \frac{\ln 2}{t_{1/2}}$$
$$t = \frac{1}{\lambda} \ln\!\left(1 + \frac{D}{P}\right)$$

D radiogenic daughter atoms, P remaining parent atoms. Half-lives: ^{14}C 5730 yr, ^{40}K 1.25 Gyr, ^{235}U 704 Myr, ^{238}U 4.47 Gyr, ^{87}Rb 49 Gyr

5.2 Geological Time Scale

Earth 4.54 Ga; Phanerozoic from 538.8 Ma: Paleozoic 538.8 to 251.9 Ma, Mesozoic 251.9 to 66.0 Ma, Cenozoic 66.0 Ma to today; Quaternary from 2.58 Ma, Holocene from 11.7 ka

5.3 Mohs Hardness

1 talc, 2 gypsum, 3 calcite, 4 fluorite, 5 apatite, 6 orthoclase, 7 quartz, 8 topaz, 9 corundum, 10 diamond

6 Earthquakes and Earth's Interior

6.1 Seismic Waves

$$v_P = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}}, \qquad v_S = \sqrt{\frac{\mu}{\rho}}$$
$$d = \Delta t_{S-P}\, \frac{v_P\, v_S}{v_P - v_S}$$

K bulk modulus, μ shear modulus [Pa]; S waves do not travel through liquids. In the crust $d \approx 8$ km per second of S−P delay

6.2 Magnitude and Energy

$$M_0 = \mu\, A\, D, \qquad M_w = \tfrac{2}{3}\,(\log_{10} M_0 - 9.1)$$
$$\log_{10} E \approx 1.5\, M_w + 4.8$$

M_0 seismic moment [N·m], A fault area, D average slip, E radiated energy [J]: one magnitude step is about 32 times more energy

6.3 Heat Flow and Isostasy

$$q = -k\,\frac{dT}{dz}, \qquad \frac{dT}{dz} \approx 25 \text{ to } 30 \text{ K/km}$$

$$r = h\,\frac{\rho_c}{\rho_m - \rho_c} \quad \text{(Airy isostasy)}$$

k thermal conductivity, h topography, r crustal root. Depths: Moho 5 to 70 km, core-mantle boundary 2891 km, inner core from 5150 km

7 Volcanoes

7.1 Magma Types

Basaltic: 45 to 52% SiO_2, 1000 to 1200 °C, low viscosity, effusive. Andesitic: 52 to 63% SiO_2. Rhyolitic: above 69% SiO_2, 650 to 800 °C, high viscosity, explosive

7.2 Volcanic Explosivity Index

VEI 0: under 10^4 m^3 of tephra, VEI 1: 10^4 to 10^6 m^3, then each step from VEI 2 ($\geq 10^6$ m^3) to VEI 8 (≥ 1000 km^3) is ten times more

8 Plate Tectonics

8.1 Plate Motion

$$v = \frac{d}{t}, \qquad 1 \text{ cm/yr} = 10 \text{ km/Myr}$$

Full spreading rate = twice the half rate measured on one side of a ridge

8.2 Ocean Floor Depth

$$d(t) \approx 2500 \text{ m} + 350 \text{ m} \cdot \sqrt{t/\text{Myr}}$$

Ocean floor deepens as the plate cools with age t (valid for $t \lesssim 70$ Myr)